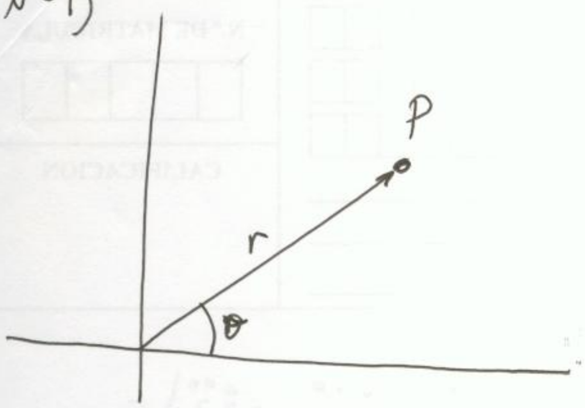
Nº 1)



$$\vec{v} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$$

$\dot{\theta} = \omega = cte.$

$$\frac{\ddot{r} - r\dot{\theta}^2}{r\ddot{\theta} + 2\dot{r}\dot{\theta}} = K \Rightarrow \frac{\ddot{r} - r\omega^2}{2\dot{r}\omega} = k \Rightarrow \ddot{r} - 2k\omega\dot{r} - \omega^2 r = 0$$

Resolvemos la E.D.O ~~con~~ C.I. $\begin{cases} r(0) = 0 \\ \text{nada mas (o bien } \theta(0) = 0 \end{cases}$

$$\lambda^2 - 2k\omega\lambda - \omega^2 = 0 \Rightarrow \lambda = k\omega \pm \sqrt{k^2\omega^2 + \omega^2} = \omega(K \pm \sqrt{1+k^2})$$

$$\lambda_1 = \omega(K + \sqrt{1+k^2})$$

$$\lambda_2 = \omega(k - \sqrt{1+k^2})$$

$$r(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t} = e^{\omega k t} \cdot (Ae^{\omega\sqrt{1+k^2}\cdot t} + Be^{-\omega\sqrt{1+k^2}\cdot t})$$

suponiendo $r(0) = 0 \quad A = -B$

$$r(t) = Ae^{\omega k t}(e^{\omega\sqrt{1+k^2}\,t} + e^{-\omega\sqrt{1+k^2}\cdot t}) = 2Ae^{\omega k t}\cosh(\omega\sqrt{1+k^2}\cdot t)$$

$$\theta(t) = \omega \cdot t + \theta_0$$

La ec. implícita es

$$\boxed{r = 2Ae^{\theta \cdot k}\cosh(\sqrt{1+k^2}\cdot\theta)}$$

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Nº2) $\vec{a} = a_T \vec{\lambda} + a_N \vec{\mu}$

$|\vec{v}| = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}$

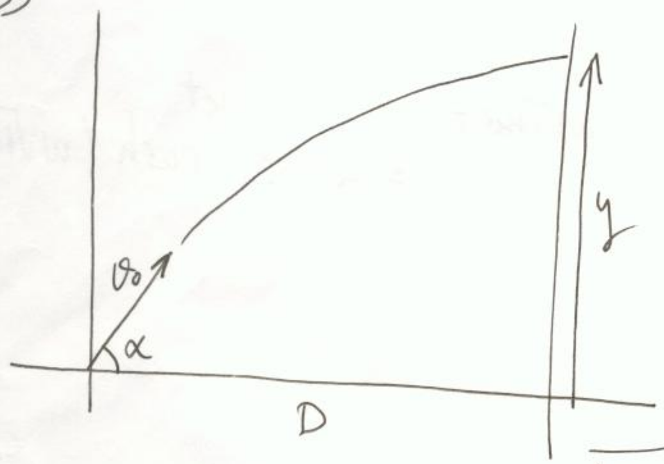
$$a_T = \frac{dv}{dt} = \frac{1}{2v} \cdot (2\dot{x}\ddot{x} + 2\dot{y}\ddot{y} + 2\dot{z}\ddot{z}) =$$

$$= \frac{\vec{v} \cdot \vec{a}}{v}$$

$$a_N = \frac{\vec{v}^2}{\rho}$$

por tanto: $\vec{a}^2 = a_T^2 + a_N^2 = \left(\frac{\vec{v} \cdot \vec{a}}{v}\right)^2 + \frac{v^4}{\rho^2}$

Nº3)



$x = v_0 \cos\alpha \cdot t$

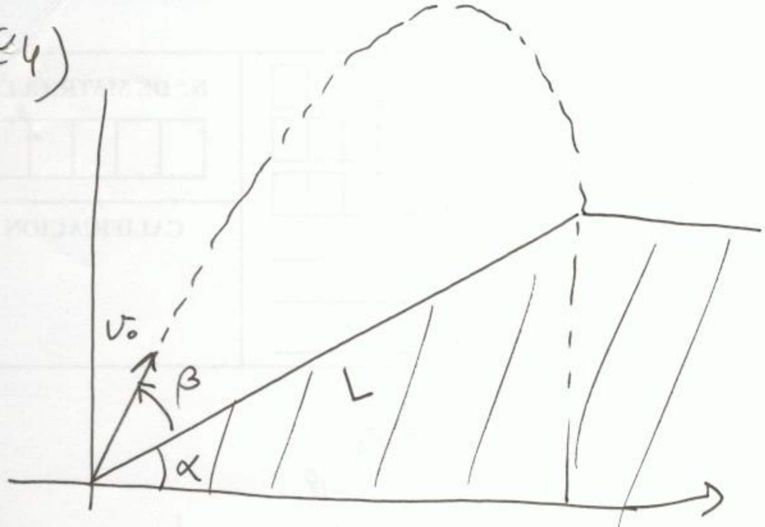
$y = v_0 \operatorname{sen}\alpha \cdot t - \frac{1}{2} g t^2$

$t = \frac{D}{v_0 \cos\alpha}$

$$y = D \operatorname{tg}\alpha - \frac{1}{2} \frac{g D^2}{v_0^2 \cos^2\alpha} = n \cdot d$$

$$\boxed{n = \operatorname{int.}\left(\frac{y}{d}\right)}$$

$N^o 4)$

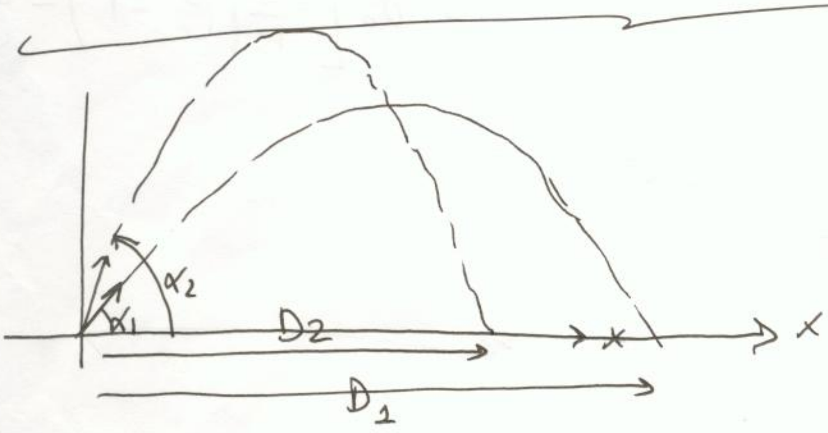


$$y = x\,tg(\beta+\alpha) - \frac{1}{2}\frac{g x^2}{v_0^2 \cos^2(\alpha+\beta)}$$

debe pasar por el punto $\left.\begin{array}{l} x = L\cos\alpha \\ y = L\,sen\,\alpha \end{array}\right\}$

$$L\,sen\,\alpha = L\cos\alpha \cdot tg(\alpha+\beta) - \frac{1}{2} g \frac{L^2\cos^2\alpha}{v_0^2\cos^2(\alpha+\beta)} \rightarrow \text{despejo } \beta$$

$N^o 5$



$x_1 = v_0 t \cos\alpha_1$

$y_1 = v_0 t\,sen\,\alpha_1 - \frac{1}{2} g t^2$

$y_1 = x_1 tg\alpha_1 - \frac{g}{2v_0^2\cos^2\alpha_1}\cdot x_1^2$

$D_1 = \frac{2v_0^2\,sen\,\alpha_1\cos\alpha_1}{g}$

$x_2 = v_0 t\cos\alpha_2$

$y_2 = v_0 t\,sen\,\alpha_2 - \frac{1}{2} g t^2$

$y_2 = x_2 tg\alpha_2 - \frac{g}{2v_0^2\cos^2\alpha_2} x_2^2$

$D_2 = \frac{2v_0^2\,sen\,\alpha_2\cos\alpha_2}{g}$

$$\Delta D = D_1 - D_2 = \frac{2v_0^2}{g}(sen\,\alpha_1\cos\alpha_1 - sen\,\alpha_2\cos\alpha_2) \rightarrow v_0$$

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Nº 6) $a = -k(v_0 + v)$ k, v_0 ctes $x(0) = 0$ $\dot{x}(0) = v_0$

$$\frac{dv}{dt} = -k(v_0+v) \Rightarrow \frac{dv}{v_0+v} = -k\,dt \Rightarrow \Big[\ln(v_0+v)\Big]_{v_0}^{v} = -kt \Rightarrow$$

$$\ln\left(\frac{v_0+v}{2v_0}\right) = -kt \Rightarrow v + v_0 = 2v_0 e^{-kt} \Rightarrow \boxed{v(t) = v_0\left(2e^{-kt} - 1\right)}$$

posición: $x(t) = x(0) + \int_0^t v(t)\,dt = \int_0^t v_0\left(2e^{-kt}-1\right)dt =$

$$= v_0\left\{\left[\frac{2e^{-kt}}{-k}\right]_0^t - t\right\} = v_0\left[-\frac{2}{k}\left(e^{-kt}-1\right) - t\right] =$$

$$= v_0\left[\frac{2}{k}\left(1-e^{-kt}\right) - t\right]$$

Nº 7) $x(t) = 2t^2 \Rightarrow \dot{x} = 4t \Rightarrow \ddot{x} = 4$

$y(t) = 4t \Rightarrow \dot{y} = 4 \Rightarrow \ddot{y} = 0$

$$\vec{\lambda} = \frac{\vec{v}}{v} = \frac{4t}{4\sqrt{1+t^2}}\vec{i} + \frac{4}{4\sqrt{1+t^2}}\vec{j} = \frac{t}{\sqrt{1+t^2}}\vec{i} + \frac{1}{\sqrt{1+t^2}}\vec{j}$$

$$\frac{d\vec{\lambda}}{dt} = \frac{\sqrt{1+t^2} - t\cdot\frac{2t}{2\sqrt{1+t^2}}}{1+t^2}\vec{i} + -\frac{1}{2}(1+t^2)^{-3/2}\cdot 2t\,\vec{j} =$$

$$= \frac{\cancel{1}+\cancel{t^2}-\cancel{t^2}}{(1+t^2)^{3/2}}\vec{i} - \frac{t}{(1+t^2)^{3/2}}\vec{j} \quad ; \quad \left|\frac{d\vec{\lambda}}{dt}\right| = \frac{1}{(1+t^2)^3} + \frac{t^2}{(1+t^2)^3}$$

$$= \frac{1}{(1+t^2)^2}$$

Nº7) CONT.

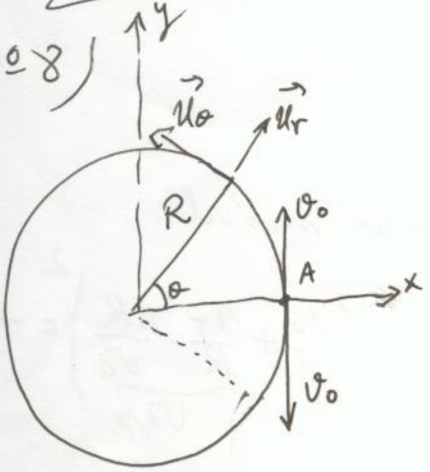
luego el radio de curvatura es

$$\rho = \frac{v}{\left|\frac{d\vec{\tau}}{dt}\right|} = \frac{4\sqrt{1+t^2}}{1/(1+t^2)^2} = 4\cdot(1+t^2)^{5/2}$$

y la aceleración tangencial

$$a_T = \frac{v^2}{\rho} = \frac{16(1+t^2)}{4(1+t^2)^{5/2}} = 4\cdot(1+t^2)^{-3/2}$$

Nº 8)



$$\vec{v} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$$

Si $r = cte = R$

$$\vec{v} = R\dot{\theta}\,\vec{u}_\theta$$

a_T cte

$$\vec{a} = \underbrace{-R\dot{\theta}^2}_{a_N}\vec{u}_r + R\ddot{\theta}\,\vec{u}_\theta$$

Llamamos $a_T = R\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{a_T}{R} \Rightarrow \theta(t) = \theta(0) + \dot{\theta}(0)t + \frac{1}{2}\frac{a_T}{R}t^2$

$$\dot{\theta}(0) = \omega_0$$

$$\theta(0) = \theta_0$$

a)

MOVIL-1 $\omega_0 = \frac{v_0}{R}$, $\theta(0) = 0 \Rightarrow \theta_1(t) = \frac{v_0}{R}t + \frac{1}{2}\frac{a_T}{R}t^2$

MOVIL-2 $\omega_0 = -\frac{v_0}{R}$, $\theta(0) = 2\pi \Rightarrow \theta_2(t) = 2\pi - \frac{v_0}{R}t + \frac{1}{2}\frac{a_T}{R}t^2$

OJO

<u>ENCUENTRO</u>: $\theta_1(t_c) = \theta_2(t_c)$ $\quad \frac{v_0}{R}t_c + \cancel{\frac{1}{2}\frac{a_T}{R}t_c^2} = 2\pi - \frac{v_0}{R}t_c + \cancel{\frac{1}{2}\frac{a_T}{R}t_c^2}$

$$\dot{\theta}_2(t_c) = -\frac{v_0}{R} + \frac{a_T}{R}t = 0$$

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de donde $t_c = \frac{\pi . R}{v_0}$ y $a_T = \frac{v_0}{t_c} = \frac{v_0^2}{\pi . R}$

b) $a = (a_r^2 + a_\theta^2)^{1/2}$

$a_r = -R\dot{\theta}^2$

$a_\theta = R\ddot{\theta} = \frac{v_0^2}{\pi R}$

queda expresar la aceleración radial para cada móvil.

MOVIL-1. $a_{r_1} = -R\dot{\theta}_1^2 = -R\left(\frac{v_0}{R} + \frac{a_T}{R}t_c\right)^2 = -R\left(\frac{v_0}{R} + \underbrace{\frac{a_T}{R}\cdot\frac{\pi R}{v_0}}_{v_0/R}\right)^2 = -R\left(\frac{2v_0}{R}\right)^2$

$= -\frac{4v_0^2}{R}$

MOVL-2 $a_{r2} = -R\dot{\theta}_2^2 = 0$ por el enunciado

luego $a_1^2 = (a_{r_1}^2 + a_{\theta_1}^2) = \left(\frac{4v_0^2}{R}\right)^2 + \left(\frac{v_0^2}{\pi R}\right)^2$

$a_2^2 = a_{\theta_2}^2 = \left(\frac{v_0^2}{\pi R}\right)^2$