FISICA I — SOLUCIONES — 24/I/12

$M_1 = L/r \rightarrow 1^{er}$ MONOMIO PI

1) $\Pi = V^{a_1} \rho^{a_2} r^{a_3} \eta^{a_4} = (LT^{-1})^{a_1} (L^{-3}MT^{-3})^{a_2} L^{a_3} (L^{-1}MT^{-1})^{a_4} = L^{a_1 - a_2 + a_3 - a_4} M^{a_2 + a_4} T^{-a_1 - 2a_2 - a_4} = L^0 M^0 T^0$

$[V] = LT^{-1}$
$[\rho] = L^{-3}MT^{-3}$
$[r] = L$
$[\eta] = L^{-1}MT^{-1}$

L: $a_1 - a_2 + a_3 - a_4 = 0$
M: $a_2 + a_4 = 0$
T: $-a_1 - 2a_2 - a_4 = 0$

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ -1 & -2 & 0 & 1 \end{pmatrix}$$

$h = 3$; $\hat{i} = 4 - 3 = 1$ MONOMIO Π

di $a_1 = 1 \rightarrow \begin{cases} a_2 = -1 \\ a_3 = -1 \\ a_4 = +1 \end{cases}$

$\Pi_2 = V^1 \rho^{-1} r^{-1} \eta^{+1} = \frac{V\eta}{\rho r} \Rightarrow F(\Pi_1, \Pi_2) = 0 \Rightarrow \Pi_2 = \varphi(\Pi_1) \rightarrow \frac{V\ell}{\rho r} = \varphi(\Pi_1) \rightarrow \boxed{V = \frac{\rho r}{\ell} \varphi(\ell/r)}$ 1,2p

MODELO	PROTOTIPO
r_M	$r_p = 5r_M$
η_M	$\eta_p = 2\eta_M$
$V_M = ?$	$V_p = 20$ m/s
ρ_M	$\rho_p = 3\rho_M$

$\frac{V_M \eta_M}{\rho_M r_M} = \frac{V_p \eta_p}{\rho_p r_p}$ → CONDICION DE SIMILITUD TOTAL

$\frac{V_M \eta_M}{\rho_M r_M} = \frac{20 \eta_M}{3\rho_M 5 r_M} \rightarrow \boxed{V_M = \frac{40}{15} = 2'6 \text{ m/s}}$ 0'5p

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2) a) $\boxed{V_B = \sqrt{2gh} = \sqrt{2g \cdot R \cdot \sin(30°)} = \sqrt{50} \text{ m/s}}$

$\sum \vec{F} = m\vec{a} \Rightarrow \sum F_n = m \cdot a_n \Rightarrow$

$N - W \cdot \cos(60°) = m \cdot V_B^2 / R$

$\boxed{N = m\left[V_B^2/R + g\cos(60°)\right] = 8400 \text{ N}}$ 0'75p

b) $V_C = \sqrt{2gR} = 10 \text{ m/s}$

$\sum \vec{F} = m \cdot \vec{a}$ $\quad -\mu_c \cdot m \cdot g = m \cdot a \Rightarrow a = -\mu_c \cdot g$

$-F_{r,c} = m \cdot a$
$N - W = 0$ $\Big\}$ $V_F^2 = V_0^2 + 2as$

$V_D^2 = 100 + 2(-0{,}2 \times 10) \times 5 = 80 \Rightarrow \boxed{V_D = 8{,}9 \text{ m/s}}$ 0'75p

c) $m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 v_1' + m_2 v_2'$

$40(8'9) + 60(-8'9) = 40 v_1' + 40 v_2'$

$\boxed{v_1' + v_2' = 0}$ (1)

$e = 0'3 = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{v_2' - v_1'}{8'9 - (-8'9)} = \frac{v_2' - v_1'}{2 \cdot 8'9}$

$\boxed{v_2' - v_1' = 5'34}$ (2) resolviendo (1) y (2) obtenemos $\boxed{v_2' = 2'67 \text{ m/s}}$, $\boxed{v_1' = -2'67 \text{ m/s}}$ 0,7

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3) a) $\sum F_x = F - F_{r,e} = M \cdot a_{cm}$

$\sum F_y = N - W = 0$

$M_{ext,cm} = R \cdot F_{r,e} = I \cdot \alpha$

$I = MR^2$, $a_{cm} = \alpha \cdot R$ $\Big\} \Rightarrow$

$\Rightarrow F = 2M a_{cm} \Rightarrow \boxed{a_{cm} = \frac{F}{2M} = \frac{15}{5} = 3 \text{ m/s}^2}$

$\boxed{F_{r,e} = M \cdot a_{cm} = 2{,}5 \times 3 = 7{,}5 \text{ N}}$ 1p

b) $F_{r,e} = \mu_{e_{max}} mg \rightarrow \mu_{e_{max}} = \frac{7'5}{25 \cdot 10} = 0'3$ 0'3p

c) $E_c = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} M R^2 \left(\frac{v_{cm}}{R}\right)$

$v_{cm} = v_0 + a_{cm} \cdot t = 3 \times 2 = 6 \text{ m/s}$

$\boxed{E_c = M v_{cm}^2 = 2{,}5 \times 6^2 = 90 \text{ J}}$ 0'6

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4) a) $Q = 150 \text{ l/min} = \frac{150 \times 10^{-3} \text{ m}^3}{60 \text{ s}} = 2{,}5 \times 10^{-3} \frac{\text{m}^3}{\text{s}}$

$Q = v \cdot S \Rightarrow v_3 = \frac{2{,}5 \times 10^{-3}}{5 \times 10^{-4}} = 5 \text{ m/s}$

$v_2 \cdot S_2 = v_3 \cdot S_3 \Rightarrow v_2 = v_3/2 = 2{,}5 \text{ m/s}$

Bernoulli: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g z_3$

$P_1 = 1{,}2 \text{ atm} = 1{,}2 \times 10^5 \text{ Pa}$ $\quad z_1 = 0$, $z_3 = H$, $v_1 \approx 0$

$P_3 = 1 \text{ atm} = 10^5 \text{ Pa}$

$0{,}2 \times 10^5 - \frac{1}{2} \times 10^3 \times 5^2 = 10^3 \times 10 \times H$

$\boxed{H = 0{,}75 \text{ m}}$ 1'2p

b) $P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g z_3$

$z_2 = z_3$

$\boxed{P_2 = P_3 + \frac{1}{2}\rho(v_3^2 - v_2^2) = 109{,}375 \text{ kPa}}$ 0'5p