1º EXAMEN FISICA I — SOLUCIONES — 9-II-04

$M = \alpha^{a_1} F^{a_2} I^{a_3} R^{a_4} = (T^{-2})^{a_1} (LMT^{-2})^{a_2} (L^2M)^{a_3} (L)^{a_4} = L^0 M^0 T^0$

$[R] = L^1 M^0 T^0$
$[I] = L^2 M T^0$
$[F] = LMT^{-2}$
$[\alpha] = L^0 M^0 T^{-2}$

$$\begin{cases} L: & a_2 + 2a_3 + a_4 = 0 \\ M: & a_2 + a_3 = 0 \\ T: & -2a_1 - 2a_2 = 0 \end{cases} \quad \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ -2 & -2 & 0 & 0 \end{pmatrix} \quad h = 3 \quad i = 4 - 3 = 1$$

haciendo $a_1 = 1$ y resolviendo, obtenemos $a_2 = -1$; $a_3 = 1$ y $a_4 = -1 \Rightarrow \Pi = \alpha^1 F^{-1} I^1 R^{-1}$

$F(\Pi) = 0 \Rightarrow \Pi = CTE \Rightarrow \frac{\alpha I}{FR} = CE \Rightarrow \boxed{\alpha = CTE \frac{F \cdot R}{I}}$

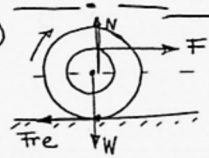
MODELO	PROTOTIPO
α_M	α_P
$I_M = \frac{I_P}{10}$	I_P
$R_M = \frac{R_P}{10}$	R_P
$F_M = F_P$	F_P

$\boxed{\Pi_{MODELO} = \Pi_{PROTOTIPO}}$ <CONDICION DE SIMILITUD COMPLETA>

$$\frac{\alpha_M I_M}{F_M R_M} = \frac{\alpha_P I_P}{F_P R_P} \Rightarrow \boxed{\frac{\alpha_M}{\alpha_P} = \frac{F_M}{F_P} \frac{R_M}{R_P} \frac{I_P}{I_M} = 1 \cdot \frac{2}{10} = \frac{1}{5}}$$

($R_P = 10 R_M$, $I_P = 2 I_M$)

3º

a) 

$\Sigma \vec{F}_{ext} = m \cdot \vec{a}_{cm} \Rightarrow \begin{cases} F - Fre = m\, a_{cm} \\ W - N = 0 \end{cases}$

$I \vec{M}_{cm} = I_{cm} \cdot \vec{\alpha} \Rightarrow F \cdot r + Fre \cdot R = \frac{1}{2} m R^2 \cdot \alpha$

$a_{cm} = \alpha \cdot R$

$\Rightarrow F\left(1 + \frac{r}{R}\right) = \left(m + \frac{1}{2} m\right) a_{cm}$

a) $\boxed{a_{cm} = \frac{2}{3} \cdot \frac{F}{m} \cdot \left(1 + \frac{r}{R}\right) = \frac{2}{3} \times \frac{200}{25} \cdot \left(1 + \frac{0.1}{0.4}\right) = 6.6\ m/s^2}$

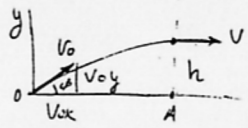
b) $E_C = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \cdot \frac{1}{2} m R^2 \left(\frac{v_{cm}}{R}\right)^2$

$= \frac{3}{4} m v_{cm}^2$

$v_{cm}^2 = v_0^2 + 2 a_{cm} s = 2 \times \frac{20}{3} \times 2 = \frac{80}{3}\ (m/s)^2 \Rightarrow$

$\boxed{E_C = 500\ J}$

2º

a) 

$V_{0x} = V_0 \cos\phi = 6\sqrt{3}\ m/s$
$V_{0y} = V_0 \operatorname{sen}\phi = 6\ m/s$

$V_y = V_{0y} - gt = 6 - 10t = 0 \Rightarrow \boxed{t = 0.6\ s}$

$\overline{OA} = x = x_0 + V_{0x} \cdot t = 6\sqrt{3}\, t \Rightarrow \boxed{\overline{OA} = 6.23\ m}$

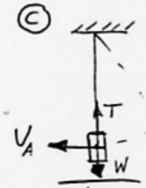
$h = y = y_0 + V_{0y} t - \frac{1}{2} g t^2 = 6 - 5t^2 \Rightarrow \boxed{h = 1.8\ m}$

b) Proyectil:

Antes del choque: $V_{1i} = V_{0x} = 6\sqrt{3}\ m/s$

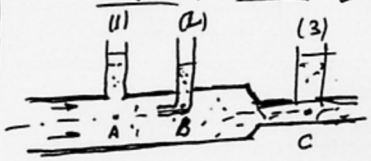
Choque: $\begin{cases} m V_{1i} + 0 = m_2 V_{1F} + m_2 V_{2F} \\ e = -\frac{V_{2F} - V_{1F}}{V_{2i} - V_{1i}} = 1 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} 0.5\, V_{2F} + 0.05\, V_{1F} = 0.05 \times 6\sqrt{3} \\ V_{2F} - V_{1F} = V_{1i} = 6\sqrt{3} \end{cases} \Rightarrow \boxed{\begin{array}{l} V_{2F} = 1.89\ m/s \\ V_{1F} = -8.5\ m/s \end{array}}$

c) 

$T - W = m \frac{V_A^2}{\ell}$ $\quad V_A = V_{2F} = 1.89\ m/s$

$\boxed{T = m\left(g + \frac{V_A^2}{\ell}\right) = 6.19\ N}$



CONTINUIDAD $\quad g \approx 10\ m/s^2$

$S_1 V_A = S_2 V_C = 5 \cdot 10^{-3}\ m^3/s$

$V_A = \frac{5 \cdot 10^{-3}}{4 \cdot 10^{-4}} = 12.5\ m/s$; $V_C = \frac{5 \cdot 10^{-3}}{10 \cdot 10^{-4}} = 5\ m/s$

(A-C) BERNOULLI

$P_A + \frac{1}{2} \rho V_A^2 + \rho g z_A = P_C + \frac{1}{2} \rho V_C^2 + \rho g z_C \Rightarrow P_A - P_C = \frac{1}{2} \rho (V_C^2 - V_A^2)$

$z_A = z_C$ $\quad P_A = P_0 + \rho g h_1$ $\quad P_C = P_0 + \rho g h_3$ $\quad P_0$: P. atmosférica

$P_0 - P_C = \rho g (h_1 - h_3) = \frac{1}{2} \rho (V_C^2 - V_A^2)$

$\boxed{h_1 - h_3 = \frac{V_C^2 - V_A^2}{2g} = \frac{18.75}{16}\ m = 1.09\ m}$

(A-D) BERNOULLI

$P_A + \frac{1}{2} \rho V_A^2 + \rho g z_A = P_D + \frac{1}{2} \rho V_B^2 + \rho g z_B \Rightarrow P_B - P_A = \frac{1}{2} \rho V_A^2 = \rho g (h_2 - h_1)$

$z_A = z_B$ $\quad V_B = 0$ $\quad P_B = P_D + \rho g h_2$ $\quad P_B - P_A = \rho g (h_2 - h_1)$ $\Rightarrow h_2 - h_1 = \frac{V_A^2}{2g} = 1.8\ cm$