

ϕ ESCALAR $\phi(u_1, u_2, u_3)$

$$\vec{A} = A_1 \vec{e}_1 + A_2 \vec{e}_2 + A_3 \vec{e}_3$$

COORDENADAS CURVILINEAS

$$a) \vec{\nabla} \phi = \frac{1}{h_1} \frac{\partial \phi}{\partial u_1} \vec{e}_1 + \frac{1}{h_2} \frac{\partial \phi}{\partial u_2} \vec{e}_2 + \frac{1}{h_3} \frac{\partial \phi}{\partial u_3} \vec{e}_3$$

$$b) \vec{\nabla} \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u_1} (h_2 h_3 A_1) + \frac{\partial}{\partial u_2} (h_1 h_3 A_2) + \frac{\partial}{\partial u_3} (h_1 h_2 A_3) \right]$$

$$c) \vec{\nabla} \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \vec{e}_1 & h_2 \vec{e}_2 & h_3 \vec{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

$$d) \vec{\nabla}^2 \phi = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial \phi}{\partial u_1} \right) + \frac{\partial}{\partial u_2} \left(\frac{h_1 h_3}{h_2} \frac{\partial \phi}{\partial u_2} \right) + \frac{\partial}{\partial u_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \phi}{\partial u_3} \right) \right]$$

SISTEMAS COORDENADOS MAS COMUNES

1) CARTESIANAS:

2) CILINDRICAS

3) ESFERICAS

$$\begin{array}{lll} u_1 = x, & u_2 = y, & u_3 = z & h_1 = h_2 = h_3 = 1 \\ u_1 = \rho & u_2 = \theta, & u_3 = z & h_1 = 1, h_2 = \rho, h_3 = 1 \\ u_1 = r & u_2 = \theta, & u_3 = \varphi & h_1 = 1, h_2 = r, h_3 = r \sin \theta \end{array}$$

Σ : CILINDRICAS

$$\vec{\nabla} \phi = \frac{\partial \phi}{\partial \rho} \vec{u}_\rho + \frac{1}{\rho} \frac{\partial \phi}{\partial \theta} \vec{u}_\theta + \frac{\partial \phi}{\partial z} \vec{u}_z$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{\partial}{\partial \theta} (A_\theta) + \frac{\partial}{\partial z} (\rho A_z) \right] =$$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}^2 \phi = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} \left(\rho \frac{\partial \phi}{\partial \rho} \right) + \frac{\partial}{\partial \theta} \left(\frac{1}{\rho} \frac{\partial \phi}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(\rho \frac{\partial \phi}{\partial z} \right) \right]$$