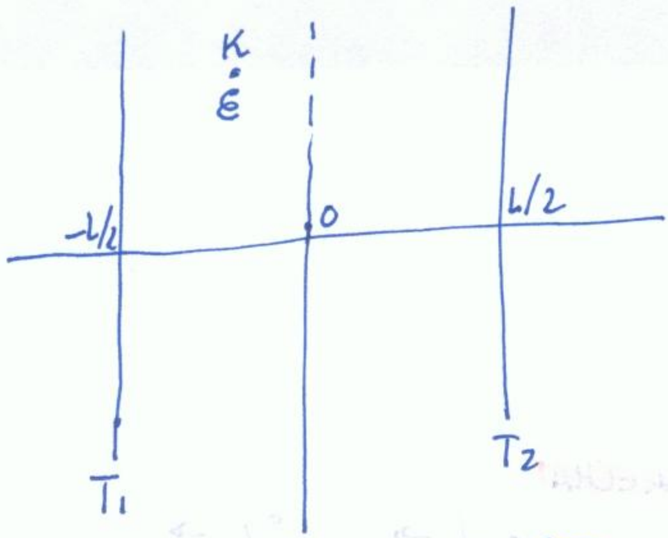
CONDUCCION 1-D EN CARTESIANAS CON FUENTES INTERNAS



$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) = -\dot{\varepsilon}$$

$$T(-L/2) = T_1$$

$$T(L/2) = T_2$$

$$K\frac{dT}{dx} = -\dot{\varepsilon}x + C \Rightarrow \frac{dT}{dx} = -\frac{\dot{\varepsilon}}{K}x + \frac{C}{K}$$

$$\int_{T_1}^{T(x)} dT = \int_{-L/2}^{x}\left(-\frac{\dot{\varepsilon}}{K}x + \frac{C}{K}\right)dx = \left.-\frac{\dot{\varepsilon}}{2K}x^2 + \frac{C}{K}x\right]_{-L/2}^{x} =$$

$$T(x) - T_1 = -\frac{\dot{\varepsilon}}{2K}\left(x^2 - L^2/4\right) + \frac{C}{K}\left(x + L/2\right)$$

Calculo de la constante C :

$$\int_{T_1}^{T_2} dT = \frac{C}{K}\,2\frac{L}{2} \Rightarrow C = \frac{K}{L}\left(T_2 - T_1\right)$$

con lo que el campo de temperaturas es :

$$\boxed{T(x) = T_1 - \frac{\dot{\varepsilon}}{2K}\left(x^2 - L^2/4\right) + \frac{T_2 - T_1}{L}\left(x + L/2\right)}$$

En forma parabólica: $T(x) = \underbrace{-\frac{\dot{\varepsilon}}{2K}}_{a}x^2 + \underbrace{\frac{T_2 - T_1}{L}}_{b}x + \underbrace{T_1 + \frac{\dot{\varepsilon}}{2K}\frac{L^2}{4} + \frac{T_2 - T_1}{2}}_{c}$

N: Si $T_1 = T_2$ $\boxed{T(x) = -\frac{\dot{\varepsilon}}{2K}x^2 + \frac{\dot{\varepsilon}L^2}{8K} + T_1}$

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Apellidos ______
Nombre ______ Nº de Matrícula ______
Asignatura ______
Curso ______ Grupo ______ Plan ______ Fecha ______

Calculos con el flujo

FLUJO A TRAVES DE LA PARED DERECHA

a) $\vec{q}'' = -k \vec{\nabla} T \Big|_{x=\frac{L}{2}} = -k \frac{dT}{dx}\Big|_{x=\frac{L}{2}} \vec{i} = +\dot{\varepsilon} x \Big|_{x=L/2} \cdot \vec{i} = +\dot{\varepsilon} \frac{L}{2} \vec{i}$

$\phi_{\rightarrow} = \vec{q}'' \cdot \underbrace{\vec{S}}_{S \cdot \vec{i}} = +\dot{\varepsilon} \frac{L}{2} \cdot S$

FLUJO A TRAVES DE LA PARED IZQUDA.

b) $\vec{q}'' = -k \vec{\nabla} T \Big|_{x=-L/2} = -k \frac{dT}{dx}\Big|_{x=-L/2} \vec{i} = -\dot{\varepsilon} x \Big|_{x=-L/2} \cdot \vec{i} = -\dot{\varepsilon} \frac{L}{2} \vec{i}$

$\phi_{\leftarrow} = \vec{q}'' \cdot \underbrace{\vec{S}}_{-S \vec{i}} = \dot{\varepsilon} \frac{L}{2} \cdot S$

FLUJO NETO: $\phi = \phi_{\rightarrow} + \phi_{\leftarrow} = \dot{\varepsilon} L \cdot S = \dot{\varepsilon} \cdot V$ en <u>watios</u>

¿Qué ocurre si $T_1 \neq T_2$?

$$\frac{dT}{dx} = -\frac{\overset{\circ}{\varepsilon}}{k}x + \frac{T_2 - T_1}{L}$$

Flujo a la derecha ...

$$\vec{q}_d'' = -k\frac{dT}{dx}\vec{i}\Big|_{x=\frac{L}{2}} = \Big[\overset{\circ}{\varepsilon}\frac{L}{2} + \underbrace{\frac{K}{L}(T_1 - T_2)}_{\phi/S \text{ (sin fuente)}}\Big]\vec{i}$$

$$\phi_{\rightarrow} = \vec{q}_d'' \cdot S \cdot \vec{i} = \dot{\varepsilon}\frac{L}{2}S + \frac{T_1 - T_2}{L/kS}$$

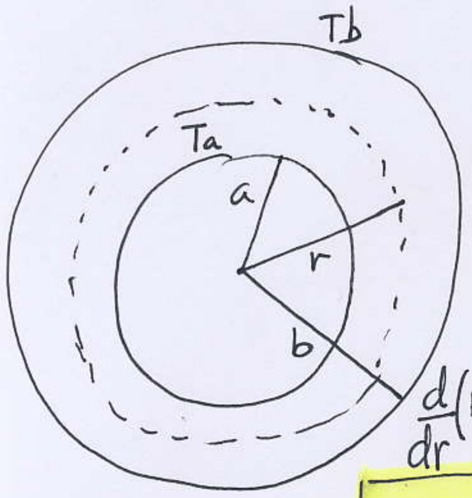
y el flujo a la izqda.

$$\vec{q}_i'' = -k\frac{dT}{dx}\Big|_{x=-L/2}\vec{i} = \Big[-\overset{\circ}{\varepsilon}\frac{L}{2} + \frac{K}{L}(T_1 - T_2)\Big]\vec{i}$$

$$\phi_{\leftarrow} = \vec{q}_i'' \cdot S(-\vec{i}) = \overset{\circ}{\varepsilon}\frac{L}{2}S - \frac{K}{L}(T_1 - T_2)S$$

FLUJO NETO: $\phi = \phi_{\rightarrow} + \phi_{\leftarrow} = \dot{\varepsilon} L \cdot S$ igual que antes

1) CONDUCCION CILINDRICA CON FUENTES $\dot{\varepsilon}$



$$\frac{1}{r}\frac{d}{dr}\left(k r \frac{dT}{dr}\right) = -\dot{\varepsilon}$$

$$T(a) = T_a$$
$$T(b) = T_b$$

$$\frac{d}{dr}\left(k r \frac{dT}{dr}\right) = -\dot{\varepsilon} r \Rightarrow \frac{dT}{dr} = -\frac{\dot{\varepsilon} r}{2k} + \frac{C}{kr}$$

$$\boxed{T(r) = -\frac{\dot{\varepsilon} r^2}{4k} + \frac{C}{k}\ln r + D}$$

$$T_a = -\frac{\dot{\varepsilon} a^2}{4k} + \frac{C}{k}\ln a + D$$

$$T_b = -\frac{\dot{\varepsilon} b^2}{4k} + \frac{C}{k}\ln b + D$$

$$T_a - T_b = -\frac{\dot{\varepsilon}}{4k}(a^2 - b^2) + \frac{C}{k}\ln\frac{a}{b}$$

$$\frac{C}{k} = \frac{T_a - T_b + \dot{\varepsilon}/4k\,(a^2 - b^2)}{\ln a/b}$$

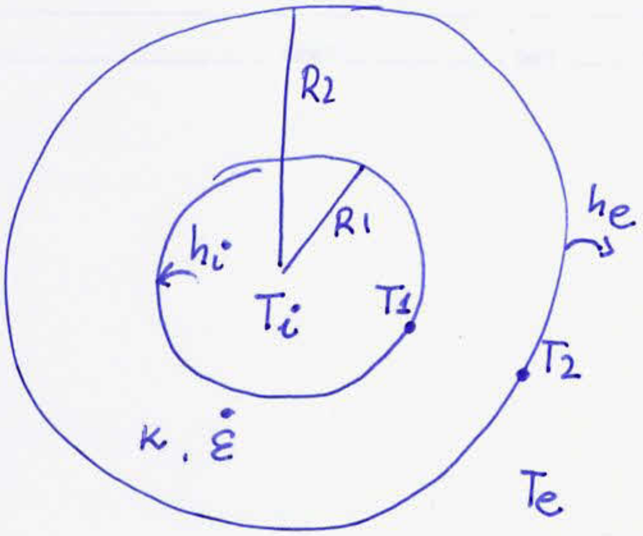
$$D = T_a + \frac{\dot{\varepsilon} a^2}{4k} - \frac{C}{k}\ln a$$

$$\boxed{T(r) = -\frac{\dot{\varepsilon}(r^2 - a^2)}{4k} + \frac{T_a - T_b + \dot{\varepsilon}/4k\,(a^2 - b^2)}{\ln a/b}\ln\frac{r}{a} + T_a}$$

(*) Si $T_a = T_b$ $\quad T(r) = -\frac{\dot{\varepsilon}}{4k}(r^2 - a^2) + \frac{\dot{\varepsilon}/4k\,(a^2 - b^2)}{\ln a/b}\cdot\ln\frac{r}{a} + T_a$

puede comprobarse que $T(r=a) = T_a$ y $T(r=b) = T_a = T_b$

CONDUCCION CILINDRICA CON FUENTES $\dot{\varepsilon}$ Y CONDICIONES DE CONTORNO NEUMANN (O MIXTAS)



$$\frac{1}{r}\frac{d}{dr}\left(k\frac{dT}{dr}\right) = -\dot{\varepsilon}$$

$$-k\left.\frac{dT}{dr}\right|_{r=R_1} = h_i(T_i - T_1)$$

$$-k\left.\frac{dT}{dr}\right|_{r=R_2} = h_e(T_2 - T_e)$$

La solución general es $\boxed{T(r) = -\frac{\dot{\varepsilon} r^2}{4K} + \frac{C}{k}\ln r + D}$

donde las constantes C y D se obtienen ahora mediante.

$$-k\left.\frac{dT}{dr}\right|_{r=R_1} = \left.\frac{\dot{\varepsilon}\cdot r}{2} - \frac{C}{r}\right|_{r=R_1} = h_i(T_i - T(R_1))$$

$$-k\left.\frac{dT}{dr}\right|_{r=R_2} = \left.\frac{\dot{\varepsilon} r}{2} - \frac{C}{r}\right|_{r=R_2} = h_e(T(R_2) - T_e)$$

Es decir, hay que calcular C y D a partir de las ecuaciones

$$\boxed{\begin{aligned} \frac{\dot{\varepsilon} R_1}{2} - \frac{C}{R_1} &= h_i\left(T_i + \frac{\dot{\varepsilon} R_1^2}{4K} - \frac{C}{k}\ln R_1 - D\right) \\ \frac{\dot{\varepsilon} R_2}{2} - \frac{C}{R_2} &= h_e\left(-\frac{\dot{\varepsilon} R_2^2}{4K} + \frac{C}{k}\ln R_2 + D - T_e\right) \end{aligned}}$$

Algo más sencillo queda si en lugar de dos condiciones Neumann, tengo una Dirichlet en el interior y una Neumann en el exterior, es decir,

$$\begin{cases} T(r)\Big|_{r=R_1} = T_1 \\ -k\dfrac{dT}{dr}\Big|_{r=R_2} = h_e\left(T(R_2)-T_e\right) \end{cases}$$

En este caso el sistema de ecuaciones queda

$$\boxed{\begin{cases} -\dfrac{\mathring{\varepsilon} R_1^2}{4k} + \dfrac{C}{k}\ln R_1 + D = T_1 \\ \dfrac{\mathring{\varepsilon} R_2}{2} - \dfrac{C}{R_2} = h_e\left(-\dfrac{\mathring{\varepsilon} R_2^2}{4k} + \dfrac{C}{k}\ln R_2 + D - T_e\right) \end{cases}}$$

Como caso particular, se estudian las condiciones de contorno Neumann

$$\begin{cases} -k\dfrac{dT}{dr}\Big|_{r=R_1} = 0 \\ -k\dfrac{dT}{dr}\Big|_{r=R_2} = h_e\left(T(R_2)-T_e\right) \end{cases}$$

El sistema de ecuaciones para encontrar C y D queda.

$$\boxed{\begin{array}{l} \dfrac{\mathring{\varepsilon} R_1}{2} - \dfrac{C}{R_1} = 0 \\ \dfrac{\mathring{\varepsilon} R_2}{2} - \dfrac{C}{R_2} = h_e\left(-\dfrac{\mathring{\varepsilon} R_2^2}{4k} + \dfrac{C}{k}\ln R_2 + D - T_e\right) \end{array}}$$

y las soluciones para C y D son:

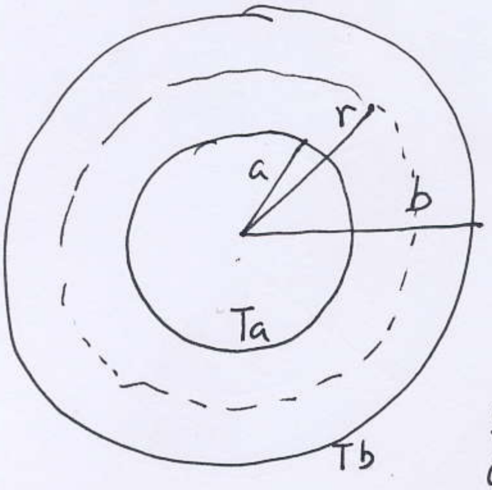
$$C = \frac{\dot{\varepsilon} R_1^2}{2}$$

$$D = \frac{\dot{\varepsilon} R_2}{2he} - \frac{\dot{\varepsilon} R_1^2}{2he R_2} + \frac{\dot{\varepsilon} R_2^2}{4K} - \frac{\dot{\varepsilon} R_1^2}{2k} \ln R_2 + T_e$$

con lo que el campo de temperaturas en la capa cilíndrica queda finalmente

$$\boxed{T(r) = -\frac{\dot{\varepsilon}}{4K}\left(r^2 - R_2^2\right) + \frac{\dot{\varepsilon} R_1^2}{2k} \ln \frac{r}{R_2} + \frac{\dot{\varepsilon}}{2he}\left(R_2 - \frac{R_1^2}{R_2}\right) + T_e}$$

2) CONDUCCION ESFERICA CON FUENTES $\dot{\varepsilon}$



$$\frac{1}{r^2}\frac{d}{dr}\left(k r^2 \frac{dT}{dr}\right) = -\dot{\varepsilon}$$

$$T(a) = T_a$$
$$T(b) = T_b$$

$$\frac{d}{dr}\left(k r^2 \frac{dT}{dr}\right) = -\dot{\varepsilon} r^2 \Rightarrow \frac{dT}{dr} = -\frac{\dot{\varepsilon} r}{3k} + \frac{C}{k}\frac{1}{r^2}$$

$$\boxed{T(r) = -\frac{\dot{\varepsilon} r^2}{6k} - \frac{C}{k}\frac{1}{r} + D}$$

$$\left.\begin{aligned} T_a &= -\frac{\dot{\varepsilon} a^2}{6k} - \frac{C}{k}\frac{1}{a} + D \\ T_b &= -\frac{\dot{\varepsilon} b^2}{6k} - \frac{C}{k}\frac{1}{b} + D \end{aligned}\right\} \quad T_a - T_b = -\frac{\dot{\varepsilon}}{6k}(a^2 - b^2) - \frac{C}{k}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\frac{C}{k} = -\frac{T_a - T_b + \dot{\varepsilon}/6k\,(a^2 - b^2)}{1/a - 1/b}$$

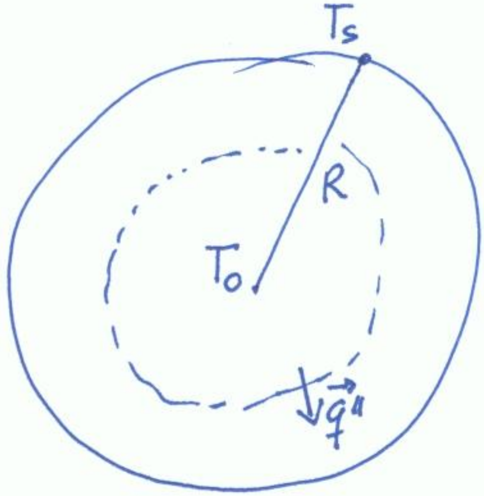
$$D = T_a + \dot{\varepsilon} a^2/6k + \frac{C}{k}\frac{1}{a}$$

$$\boxed{T(r) = -\frac{\dot{\varepsilon}}{6k}(r^2 - a^2) + \frac{T_a - T_b + \dot{\varepsilon}/6k\,(a^2 - b^2)}{1/a - 1/b}\left(\frac{1}{r} - \frac{1}{a}\right) + T_a}$$

(*) Si $T_a = T_b$

$$\boxed{T(r) = -\frac{\dot{\varepsilon}}{6k}(r^2 - a^2) + \frac{\dot{\varepsilon}/6k\,(a^2 - b^2)}{1/a - 1/b}\left(\frac{1}{r} - \frac{1}{a}\right) + T_a}$$

ESFERA MACIZA CON FUENTES DE DENSIDAD CONSTANTE



$$\begin{cases} \frac{1}{r^2}\frac{d}{dr}\left(k r^2 \frac{dT}{dr}\right) = -\dot{\varepsilon} \\ T(R) = T_s \quad o \quad T(0) = T_0 \\ \left.\frac{dT}{dr}\right|_{r=0} = 0 \end{cases}$$

$$\frac{d}{dr}\left(k r^2 \frac{dT}{dr}\right) = -\dot{\varepsilon} r^2 \Rightarrow k r^2 \frac{dT}{dr} = -\frac{\dot{\varepsilon} r^3}{3} + C \Rightarrow$$

$$\frac{dT}{dr} = -\frac{\dot{\varepsilon} r}{3k} + \frac{C}{k}\frac{1}{r} \quad \text{para} \quad \left.\frac{dT}{dr}\right|_{r=0} = 0 \Rightarrow C = 0$$

por tanto. $\frac{dT}{dr} = -\frac{\dot{\varepsilon} r}{3k} \Rightarrow \boxed{T(r) = -\frac{\dot{\varepsilon} r^2}{6k} + T_0}$

tambien $T_s = -\frac{\dot{\varepsilon} R^2}{6k} + T_0 \Rightarrow \boxed{T(r) = \frac{\dot{\varepsilon}}{6k}(R^2 - r^2) + T_s}$

y $\left[q'' = -k \left.\frac{dT}{dr}\right|_{S(r)} = \frac{\dot{\varepsilon} r}{3} \right]$ y $\left[\phi = \int_{S(r)} q'' dS = \frac{\dot{\varepsilon}}{3} 4\pi r^3 \right]$

y el flujo total saliente por la superficie $S(R)$ es $\phi = \dot{\varepsilon} \cdot \frac{4}{3}\pi R^3$ que es la potencia total generada en el interior de la esfera