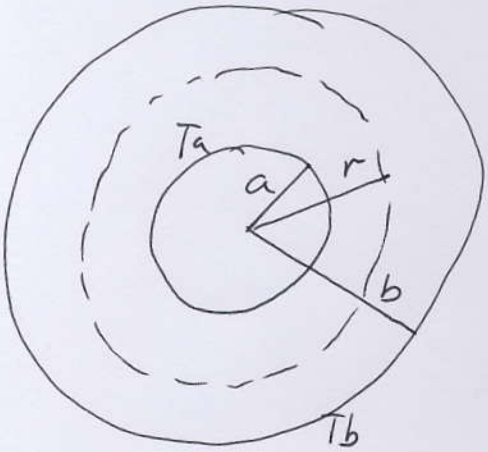
CONDUCCION EN CILINDRICAS Y ESFERICAS CON COEFICIENTE DE CONDUCTIVIDAD VARIABLE

1) CONDUCCION CILINDRICA $K(r)$



$$\left|\begin{array}{l} \frac{1}{r}\frac{d}{dr}\left(K(r)\cdot r\frac{dT}{dr}\right)=0 \\ T(a)=T_a \\ T(b)=T_b \end{array}\right.$$

$$K(r)\cdot r\cdot\frac{dT}{dr}=C \Rightarrow dT=\frac{C}{K(r)\cdot r}\cdot dr$$

i) CALCULO DE C

$$\int_{T_a}^{T_b} dT = C\int_a^b \frac{dr}{rK(r)} \Rightarrow C=\frac{T_b-T_a}{\int_a^b \frac{dr}{rK(r)}}$$

ii) CALCULO DEL PERFIL DE TEMPERATURAS $T(r)$

$$T(r)-T_a = C\cdot\int_a^r \frac{dr}{rK(r)} = \frac{T_b-T_a}{\int_a^b \frac{dr}{rK(r)}}\cdot\int_a^r\frac{dr}{rK(r)}$$

Si introduzco el factor $2\pi L$

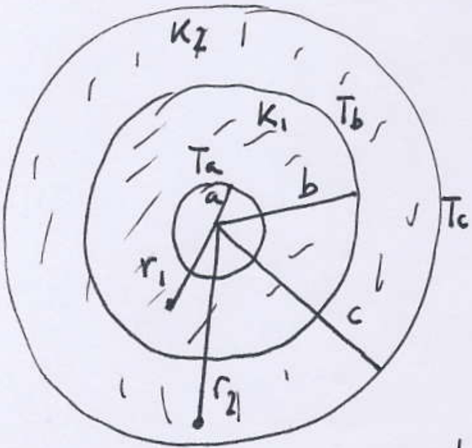
$$T_a-T(r)=\frac{T_a-T_b}{\underbrace{\frac{1}{2\pi L}\int_a^b\frac{dr}{rK(r)}}_{R_T}}\cdot\underbrace{\frac{1}{2\pi L}\int_a^r\frac{dr}{rK(r)}}_{R(r)}=\underbrace{\frac{T_a-T_b}{R_T}}_{\phi}\cdot R(r)$$

Se puede definir una resistencia termica de una capa de espesor dr y longitud L mediante

$$\boxed{dR_T=\frac{dr}{2\pi r\cdot L\ K(r)}=\frac{dr}{S(r)\cdot K(r)}}$$

APLICACIONES

a) MEDIOS CILINDRICOS COMPUESTOS



$$T_a - T(r_1) = \phi \cdot R(r_1)$$

$$\phi = \frac{T_a - T_c}{R_T}$$

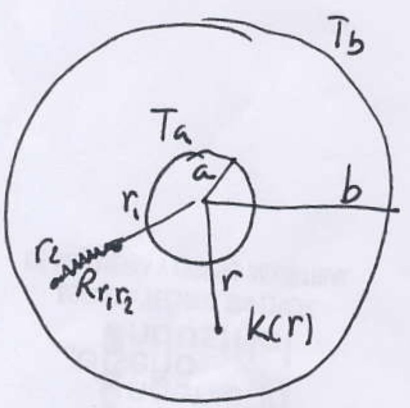
$$R_T = \int_a^c \frac{dr}{S(r)\cdot k(r)} = \frac{1}{2\pi L}\int_a^c \frac{dr}{r\,K(r)} =$$

$$\frac{1}{2\pi L}\left[\int_a^b \frac{dr}{r K_1} + \int_b^c \frac{dr}{r K_2}\right] = \frac{1}{2\pi L}\left[\frac{1}{K_1}\ln\frac{b}{a} + \frac{1}{K_2}\ln\frac{c}{b}\right]$$

y finalmente $R(r_1) = \frac{1}{2\pi L}\int_a^{r_1}\frac{dr}{r K_1} = \frac{1}{2\pi L}\cdot\frac{1}{K_1}\ln\frac{r_1}{a}$

$$T(r_1) = T_a - \frac{T_a - T_c}{R_T}\cdot R(r_1) = T_a - \frac{T_a - T_c}{\frac{1}{K_1}\ln\frac{b}{a} + \frac{1}{K_2}\ln\frac{c}{b}}\cdot\frac{1}{K_1}\ln\frac{r_1}{a}$$

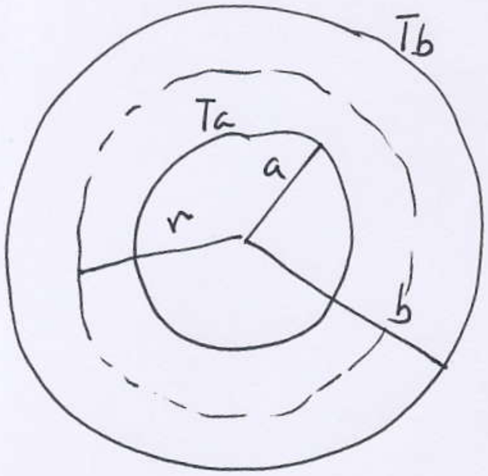
b) DIFERENCIA DE TEMPERATURA ENTRE DOS PUNTOS DE UN MEDIO CILINDRICO NO HOMOGENEO



$$T(r_1) - T(r_2) = \phi\cdot R_{r_1-r_2} = \frac{T_a - T_b}{\frac{1}{2\pi L}\int_a^b \frac{dr}{r K(r)}}\cdot \cancel{\frac{1}{2\pi L}} R_{r_1-r_2}$$

donde $R_{r_1-r_2} = \frac{1}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r K(r)}$

2) CONDUCCIÓN ESFÉRICA



$$\frac{1}{r^2}\frac{d}{dr}\left(k r^2 \frac{dT}{dr}\right) = 0$$

$$T(a) = T_a$$

$$T(b) = T_b$$

$$k r^2 \frac{dT}{dr} = C \Rightarrow dT = \frac{C}{k r^2} dr$$

i) CALCULO DE C

$$C = \frac{T_b - T_a}{\displaystyle\int_a^b \frac{dr}{k r^2}}$$

II) CALCULO DEL PERFIL DE TEMPERATURAS T(r)

$$T(r) - T_a = C \cdot \int_a^r \frac{dr}{k r^2} = \frac{T_b - T_a}{\displaystyle\int_a^b \frac{dr}{k r^2}} \cdot \int_a^r \frac{dr}{k r^2}$$

INTRODUCIENDO EL FACTOR 4π Y CAMBIANDO DE SIGNO

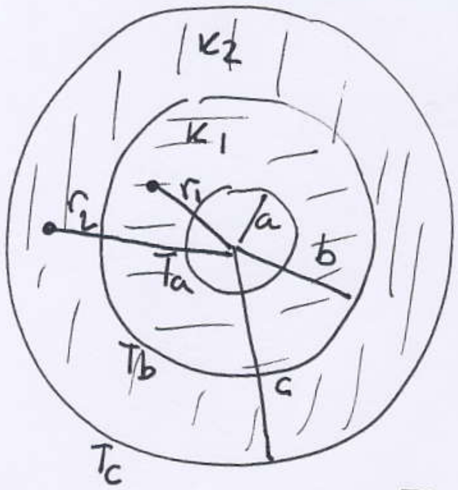
$$T_a - T(r) = \underbrace{\frac{T_a - T_b}{\displaystyle\frac{1}{4\pi}\int_a^b \frac{dr}{k r^2}}}_{\phi} \cdot \underbrace{\frac{1}{4\pi}\int_a^r \frac{dr}{k r^2}}_{R(r)} = \frac{T_a - T_b}{R_T} \cdot R(r)$$

donde $R_T = \frac{1}{4\pi}\int_a^b \frac{dr}{k r^2}$ y también se puede definir la resistencia térmica de una capa esférica de espesor dr por

$$\boxed{dR_T = \frac{dr}{4\pi r^2 K} = \frac{dr}{S(r) \cdot K(r)}}$$

APLICACIONES

MEDIOS ESFERICOS COMPUESTOS



$$T_a - T(r_2) = \phi R(r_2)$$

$$\phi = \frac{T_a - T_c}{R_T}$$

$$R_T = \int_a^c \frac{dr}{S(r)K(r)} = \int_a^c \frac{dr}{4\pi r^2 K(r)} =$$

$$= \frac{1}{4\pi}\left[\int_a^b \frac{dr}{K_1 r^2} + \int_b^c \frac{dr}{K_2 r^2}\right] = \frac{1}{4\pi}\left[\frac{1}{K_1}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{K_2}\left(\frac{1}{b}-\frac{1}{c}\right)\right]$$

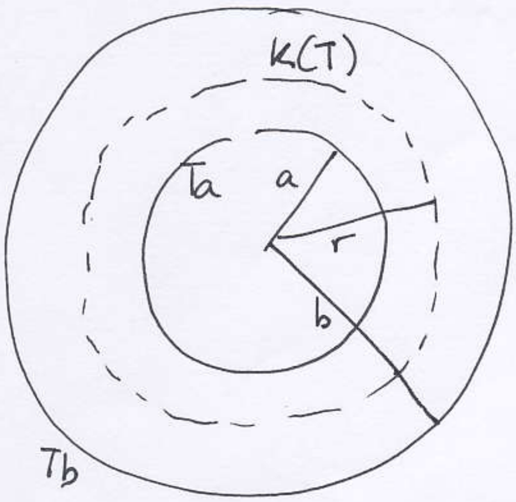
$$y \quad R(r_2) = \int_a^{r_2} \frac{dr}{4\pi r^2 K(r)} = \frac{1}{4\pi}\left[\frac{1}{K_1}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{K_2}\left(\frac{1}{b}-\frac{1}{r_2}\right)\right]$$

y finalmente

$$T(r_2) = T_a - \phi \cdot R(r_2) = T_a - \frac{T_a - T_c}{\frac{1}{K_1}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{K_2}\left(\frac{1}{b}-\frac{1}{c}\right)} \cdot \left(\frac{1}{K_1}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{K_2}\left(\frac{1}{b}-\frac{1}{r_2}\right)\right)$$

CONDUCCIÓN EN GEOMETRÍAS CILÍNDRICA Y ESFÉRICA CON COEFICIENTE DE CONDUCTIVIDAD VARIABLE CON LA TEMPERATURA

1) CILÍNDRICAS $K(T)$



$$\begin{cases} \frac{1}{r}\frac{d}{dr}\left(K(T)\cdot r\frac{dT}{dr}\right)=0 \\ T(a)=T_a \\ T(b)=T_b \end{cases}$$

$$K(T)\cdot r\frac{dT}{dr}=C \Rightarrow K(T)\,dT=\frac{C}{r}dr$$

i) CALCULO DE C

$$\int_{T_a}^{T_b}K(T)\,dT=C\int_a^b\frac{dr}{r}=C\ln b/a$$

ii) PERFIL DE TEMPERATURAS $(T(r))$

$$\int_{T_a}^{T(r)}K(T)\,dT=\frac{\int_{T_a}^{T_b}K(T)\cdot dT}{\ln b/a}\cdot\ln r/a$$

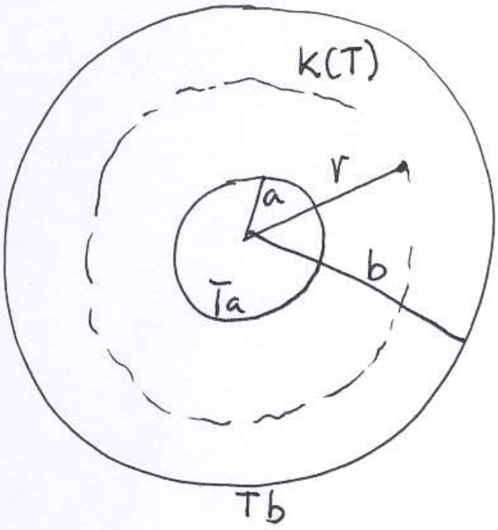
de donde

$$\ln\frac{r}{a}=\ln b/a\cdot\frac{\int_{T_a}^{T(r)}K\cdot dT}{\int_{T_a}^{T_b}K\,dT}\Rightarrow r=a\cdot e^{\ln\frac{b}{a}\cdot\frac{\int_{T_a}^{T(r)}K\,dT}{\int_{T_a}^{T_b}K\,dT}}$$

y finalmente $\boxed{r=a\cdot\left(\frac{b}{a}\right)^{\frac{\int_{T_a}^{T(r)}K\,dT}{\int_{T_a}^{T_b}K\,dT}}}$ la cual ha de ser invertida para encontrar $T=T(r)$

Ejemplo: $K(T)=C\cdot T$ $\qquad T(r)=\sqrt{\frac{T_b^2-T_a^2}{\ln b/a}\cdot\ln r/a+T_a^2}$

2) ESFERICAS $K(T)$



$$\frac{1}{r^2}\frac{d}{dr}\left(K r^2 \frac{dT}{dr}\right) = 0$$

$$T(a) = T_a$$

$$T(b) = T_b$$

$$K(T)\cdot r^2 \frac{dT}{dr} = C \Rightarrow K\cdot dT = \frac{C}{r^2}\,dr$$

i) CALCULO DE LA CONSTANTE INDETERMINADA C

$$\int_{T_a}^{T_b} K\,dT = C\int_a^b \frac{dr}{r^2} = C\left(\frac{1}{a} - \frac{1}{b}\right)$$

ii) PERFIL DE TEMPERATURAS $T(r)$

$$\int_{T_a}^{T(r)} K\,dT = \frac{\int_{T_a}^{T_b} K\,dT}{\left(\frac{1}{a} - \frac{1}{b}\right)}\cdot\left(\frac{1}{a} - \frac{1}{r}\right)$$

de donde

$$\frac{1}{a} - \frac{1}{r} = \left(\frac{1}{a} - \frac{1}{b}\right)\cdot\frac{\int_{T_a}^{T(r)} K\,dT}{\int_{T_a}^{T_b} K\,dT} \Rightarrow \boxed{\frac{1}{r} = \frac{1}{a} - \left(\frac{1}{a} - \frac{1}{b}\right)\cdot\frac{\int_{T_a}^{T(r)} K\,dT}{\int_{T_a}^{T_b} K\,dT}}$$

Finalmente habrá que invertir $r = r(T)$ para encontrar el campo de Temperaturas $T = T(r)$