EL METODO DE LAS DIFERENCIAS FINITAS PARA LA EC. DEL CALOR

$$\boxed{\begin{array}{ll} \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} = 0 & \text{EN EL DOMINIO} \\ T(x,y) = f(x,y) & \text{EN LA FRONTERA} \end{array}}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T(x+\Delta x, y) - 2T(x,y) + T(x-\Delta x, y)}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T(x, y+\Delta y) - 2T(x,y) + T(x, y-\Delta y)}{\Delta y^2}$$

Si suponemos que $\boxed{\Delta x = \beta \cdot \Delta y}$

$$\frac{1}{\Delta y^2}\left[\frac{T(x+\Delta x,y) - 2T(x,y) + T(x-\Delta x,y)}{\beta^2} + T(x,y+\Delta y) - 2T(x,y) + T(x,y-\Delta y)\right] = 0$$

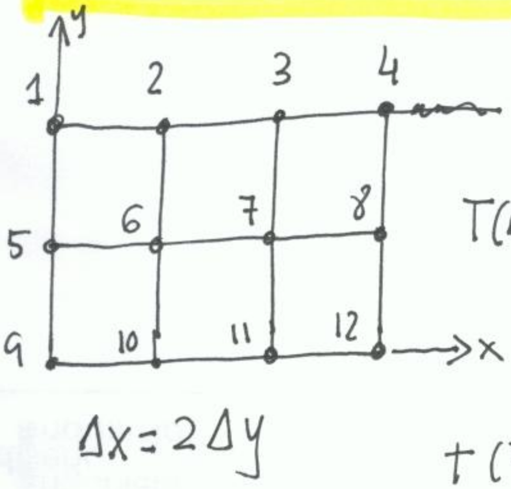
de donde

$$\left[T(x+\Delta x,y) + T(x-\Delta x,y)\right] + \beta^2\left[T(x,y+\Delta y) + T(x,y-\Delta y)\right] - 2(1+\beta^2)T(x,y) = 0$$

y finalmente

$$\boxed{T(x,y) = \frac{1}{2(1+\beta^2)}\left[T(x+\Delta x,y) + T(x-\Delta x,y)\right] + \frac{\beta^2}{2(1+\beta^2)}\left[T(x,y+\Delta y) + T(x,y-\Delta y)\right]}$$

Ej:



$$T(6) = \frac{1}{10}(T_7 + T_5) + \frac{4}{10}(T_2 + T_{10})$$

$\Delta x = 2\Delta y$

$$T(7) = \frac{1}{10}(T_8 + T_6) + \frac{4}{10}(T_3 + T_{11})$$

Si
$$\left.\begin{array}{l} T_1 = T_2 = T_3 = T_4 = 100^\circ C \\ T_9 = T_{10} = T_{11} = T_{12} = 50^\circ C \\ T_5 = T_8 = 75^\circ C \end{array}\right\}$$

$$\boxed{T_6 = T_7 = 75^\circ C}$$