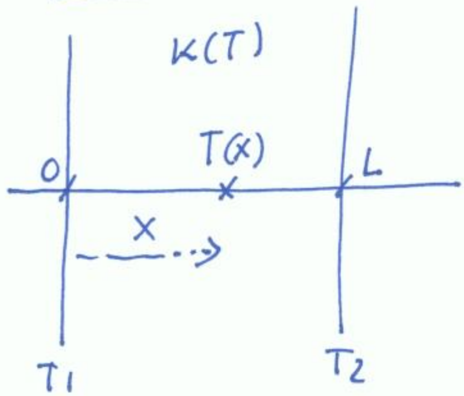
AMPLIACION DE LA ANALOGÍA ELECTRICA PARA $K=K(T)$

a) GEOMETRIA PLANA



$$\begin{cases} \dfrac{d}{dx}\left(K\dfrac{dT}{dx}\right)=0 \\ T(0)=T_1 \\ T(L)=T2 \end{cases} \Rightarrow \quad K\cdot\frac{dT}{dx}=C_1 \Rightarrow C=-\phi/S$$

$$K\,dT=-\frac{\phi}{S}\,.dx \Rightarrow \int_{T_1}^{T(x)} K\,dT=\int_0^x -\frac{\phi}{S}dx$$

de donde: $$(T(x)-T_1)\cdot\underbrace{\frac{1}{T(x)-T_1}\int_{T_1}^{T(x)} K\,dT}_{K_m(x)}=-\frac{\phi}{S}X$$

Defino una "CONDUCTIVIDAD MEDIA" $$\boxed{K_m(X)=\frac{1}{T(X)-T_1}\int_{T_1}^{T(x)} K\,dT}$$

de donde $$T(X)-T_1=-\frac{\phi.X}{S\,K_m(X)}$$

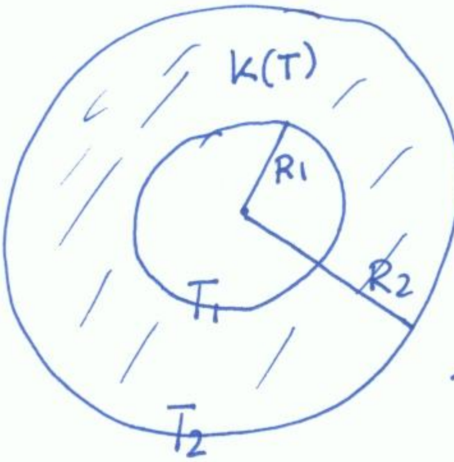
o bien

$$\boxed{\begin{aligned} T_1-T(X)&=\phi\cdot R_m(X)\\ R_m(X)&=\frac{X}{S\,K_m(X)}\\ \phi&=\frac{T_1-T2}{R_m(L)} \end{aligned}}$$

$$R_m(L)=\frac{L}{S\,K_m(L)}=$$

$$=\frac{L\,(T_2-T_1)}{S\int_{T_1}^{T_2}K.dT}$$

b) GEOMETRIA CILINDRICA



$$\begin{cases} \dfrac{1}{r}\dfrac{d}{dr}\left(K \cdot r\dfrac{dT}{dr}\right) = 0 \\ T(R_1) = T_1 \\ T(R_2) = T_2 \end{cases}$$

$$\frac{\phi}{2\pi r L} = -K\frac{dT}{dr} = -\frac{C}{r}$$

$$K\frac{dT}{dr} = -\frac{\phi}{2\pi r L} \Rightarrow K\,dT = -\frac{\phi}{2\pi L}\frac{dr}{r} \Rightarrow \int_{T_1}^{T(r)} K\,dT = -\frac{\phi}{2\pi L}\log\frac{r}{R_1}$$

Introduciendo la conductividad media...

$$(T(r)-T_1)\cdot\underbrace{\frac{1}{T(r)-T_1}\cdot\int_{T_1}^{T(r)} K\,dT}_{K_m(r)} = -\frac{\phi}{2\pi L}\log\frac{r}{R_1}$$

$$\boxed{\begin{gathered} T_1 - T(r) = \phi\cdot R_m(r) \\ R_m(r) = \frac{1}{2\pi L K_m(r)}\cdot\log\frac{r}{R_1} \\ \phi = \frac{T_1-T_2}{R_m} \end{gathered}}$$

CON. $\boxed{\begin{gathered} R_m = \frac{1}{2\pi L K_m}\cdot\log\frac{R_2}{R_1} \\ K_m = \frac{1}{T_2-T_1}\int_{T_1}^{T_2} K(T)\,dT \end{gathered}}$

c) GEOMETRIA ESFÉRICA

$$\begin{cases} \dfrac{1}{r^2}\dfrac{d}{dr}\left(K r^2 \dfrac{dT}{dr}\right) = 0 \\ T(R_1) = T_1 \\ T(R_2) = T_2 \end{cases}$$

AHORA SE CUMPLE $\dfrac{\phi}{4\pi r^2} = -K\dfrac{dT}{dr} = -\dfrac{C}{r^2} \Rightarrow C = -\dfrac{\phi}{4\pi}$

POR TANTO: $K\dfrac{dT}{dr} = \dfrac{C}{r^2} = -\dfrac{\phi}{4\pi r^2} \Rightarrow K\,dT = -\dfrac{\phi}{4\pi}\dfrac{dr}{r^2}$

$$\int_{T_1}^{T(r)} K(T)\,dT = -\frac{\phi}{4\pi}\int_{R_1}^{r}\frac{dr}{r^2} = -\frac{\phi}{4\pi}\left(\frac{1}{R_1} - \frac{1}{r}\right)$$

DE NUEVO INTRODUCIMOS LA CONDUCTIVIDAD MEDIA

$$\left(T(r) - T_1\right)\cdot \underbrace{\frac{1}{T(r)-T_1}\cdot\int_{T_1}^{T(r)} K(T)\,dT}_{K_m(r)} = -\frac{\phi}{4\pi}\left(\frac{1}{R_1} - \frac{1}{r}\right)$$

ASÍ:

$$\boxed{\begin{aligned} T_1 - T(r) &= \phi\, R_m(r) \\ R_m(r) &= \frac{1}{4\pi K_m(r)}\left(\frac{1}{R_1} - \frac{1}{r}\right) \\ \phi &= \frac{T_1 - T_2}{R_m} \end{aligned}}$$

CON

$$\boxed{\begin{aligned} R_m &= \frac{1}{4\pi K_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ K_m &= \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} K(T)\,dT \end{aligned}}$$